1. Denote by $\theta_t(x) := \frac{1}{\sqrt{4\pi t}} e^{-x^2/(4t)}$ a standard Gaussian/heat kernel, and let the measures $(\mu_n)_n \subset \mathcal{M}_1(\mathbb{R})$ be given by,

$$\mu_n(dx) := \big[a\theta_{1/n}(x-1) + (1-a)\theta_{1/n}(x+1)\big]\,dx,$$

where $a \in (0,1)$ and dx is the Lebesgue measure on $\mathbb{R}$.

   (a) Prove the LDP for μ_n (easy calculation; no Bryc or Gärtner-Ellis needed!) and sketch the rate functional.

   (b) 'Bonus' question: the Gärtner-Ellis Theorem does not apply, but can you nevertheless sketch the corresponding Λ^* that one *would* get?

2. Let $X_1, X_2, \dots$ be i.i.d. random variables in $\mathbb{R}$ with $\mathrm{Law}(X_1) = \rho \in \mathcal{M}_1(\mathbb{R})$. Assume that $\phi(t) := \mathbb{E}e^{tX_1} < \infty$ for all t, and recall that $\log\phi$ is smooth. Use Gärtner-Ellis to prove the (Cramér) LDP for the random variable $\frac{1}{n}S_n$, $S_n := \sum_{i=1}^n X_i$,