1. Let $\mathcal{X}$ be a finite set, $H : \mathcal{X} \to \mathbb{R}$ and

$$\mu_\beta(x) := \frac{1}{Z_\beta} e^{-\beta H(x)}, \qquad\qquad Z_\beta := \sum_{x\in\mathcal{X}} e^{-\beta H(x)}.$$

This is the simplest example of a Gibbs measure corresponding to the "Hamiltonian" or "energy function" H, with inverse thermodynamic temperature $\beta = 1/(k_B T)$, and k_B is the Boltzmann constant. Let $W \subset \mathcal{X}$, and define for any $A \subset \mathcal{X}$,

$$\mu_\beta^W(A) := \mu_n(A|W) = \mathrm{Prob}(X_\beta \in A | X_\beta \in W).$$

What is the large-deviation rate function $I^W : \mathcal{X} \to [0, \infty]$ corresponding to the conditioned measure μ_β^W, in the low-temperature regime $\beta \to \infty$?
Validity check: is $\inf I^W = 0$*?*