Lehrstuhl für Finanzmathematik
Fakultät für Mathematik
Technische Universität München

# Master thesis at the Chair of Mathematical Finance: *Sampling of random vectors via generative moment matching networks*

Suppose you want to sample from a target probability distribution $P$ on $\mathbb{R}^d$, which is described in terms of its characteristic function $\phi_P$. If $d = 1$ and $\phi_P$ is integrable w.r.t. the Lebesgue measure we may invert the characteristic function to obtain the corresponding probability density function $f_P$, which allows to sample from $P$. However, if $d > 1$ or $\phi_P$ is not integrable w.r.t. the Lebesgue measure the inversion of the characteristic function is much more involved or even impossible. The goal of this master thesis is to develop a novel approach for the simulation of arbitrary characteristic functions via Machine Learning techniques.

Recently, generative moment matching networks have become popular in Machine Learning. In essence, these models try to match all moments of a given dataset and samples generated from the neural network. Generative moment matching networks have been used as an alternative to generative adversarial neural networks to learn the (empirical) distribution of a given dataset. For example, [2] have used a sample from a given copula as training data to construct a neural network which imitates the samples from this copula to apply variance reduction techniques for Monte-Carlo simulation. At the heart of these algorithms is the computation of a so-called maximum mean discrepancy (MMD) metric [1] of the given dataset and the samples generated from the neural network. The theoretical foundation of these networks is based on the embedding of $P$ (i.e. a mapping $P \mapsto h_P$) into a reproducing kernel Hilbert space of functions [3, 4], where the chosen kernel uniquely determines the resulting MMD metric.

For certain MMD metrics the distance of two probability distributions $P_1$ and $P_2$ can be expressed as

$$\int_{\mathbb{R}^d} \|\phi_{P_1}(t) - \phi_{P_2}(t)\|^2 \Gamma(\mathrm{d}t),$$

where $\Gamma$ denotes a known probability measure on $\mathbb{R}^d$ and acts as a weighting function. Roughly speaking, certain MMD metrics are weighted squared distances of the characteristic function of the corresponding probability measures. In this master thesis we will train a neural network with loss function

$$\int_{\mathbb{R}^d} \|\phi_P(t) - \hat{\phi}_P(t)\|^2 \Gamma(\mathrm{d}t), \tag{1}$$

where $\hat{\phi}_P(t) = \frac{1}{n} \sum_{i=1}^n \exp(-it f_\theta(X_i))$ denotes the empirical characteristic function of $n$ i.i.d. samples $\big(f_\theta(X_i)\big)_{1\leq i\leq n}$ from the neural network $f_\theta$. If the function in (1) is differentiable w.r.t. $\theta$ one can apply the gradient-descent method to train the neural network $f_\theta$. Assuming that the trained neural network exhibits a small loss according to (1), the samples generated from the neural network closely resemble the distribution of $P$. Thus, the trained neural network may be used as a sampler for observations from $P$.

The following tasks should be completed during the master thesis

1. Provide a short introduction of kernel mean embeddings of probability distributions into reproducing kernel Hilbert spaces and the corresponding MMD metrics (no proofs).

2. Compute the gradient in (1) for simple neural network architectures or verify a stochastic gradient descent approach.

3. Implement the corresponding generative moment matching neural networks to sample from various target probability distributions $P$.

For more information students can contact *florian.brueck@tum.de*. The usual requirements for writing a masters thesis at the Chair of Mathematical Finance apply.

Are you interested in this master thesis?

We are happy to receive your application:

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*Literatur

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